

High Quality Classroom Teaching – Mathematics (Wave 1)

'For change to be successful, teachers' Mathematical beliefs, attitudes and practices need to be aligned.'

Know thy impact- What teachers do matters

General			Maths	
Instructional Strategy	Effect Size ¹	Growth ²	Instructional Strategy	Effect Size ³
Teacher estimates of achievement	1.62			
Collective teacher efficacy	1.57			
Visible Learning	1.44	+8m		
Self-reported grades	1.33			
Piagetian Programs	1.30			
Response to Intervention	1.07			
			Student think alouds	0.98 ⁴
Micro Teaching	0.90			
Classroom Discussion; Teacher Clarity	0.80			
Providing feedback	0.73	+8m	Piagetian Programs ⁵	0.73
			Providing feedback	0.71
Reciprocal Teaching; Creativity programs	0.70			
Providing formative evaluation	0.68			
			Explicit teaching (Direct Instruction)	0.65
			Peer assisted learning	0.62
Explicit teaching practices (Direct Instruction); Self-verbalisation & self-questioning; Time on task; Spaced vs Mass practice; Study Skills; Peer tutoring;	0.60		Problem-solving teaching	0.60
Mastery Learning	0.60	+5m		
Effective classroom management	0.52			
Teacher-student relationships; Questioning; Play programs	0.50			
Early Intervention	0.47	+5m		

¹ Hattie J 2009 Visible Learning; 2012; 2015

² Teaching and Learning Toolkit Australia <http://evidenceforlearning.org.au/the-toolkit/>

³ Hanover Research 2014 Best Practices in Maths Interventions <http://www.hanoverresearch.com/media/Best-Practices-in-Math-Interventions.pdf>

⁴ Hanover Research 2014 Best Practices in Maths Interventions <http://www.hanoverresearch.com/media/Best-Practices-in-Math-Interventions.pdf> p9

⁵ Hattie J 2009 p 43

Further recommendations and corresponding levels of evidence:⁶

Recommendation	Evidence Level
Screen all students at school entry ⁷ to identify those at risk for potential mathematics challenges and provide interventions to students identified as at risk.	Moderate
Correlate screening data to other data measures for relevant year levels ⁸ (eg PAT M and NAPLAN)	Moderate

Student Think Alouds (Effect size 0.98)

Student 'think-alouds' are student-centric and involve asking students to articulate their thinking processes as they solve maths problems. This is proven to be effective in engaging students in activities requiring verification, such as evaluating decisions and checking calculations⁹.

Piagetian programs (Effect size 0.73)

Hattie found that the relationship between the Piagetian stage (logical operations, concrete, formal-operational) and achievement in Maths is very high (0.73). Thus knowing the ways in which students think, and how this thinking may be constrained by their stages of development may be most important to how teachers choose material and tasks, how the concept of difficulty and challenge can be realized in different tasks and the importance of developing successive and simultaneous thinking¹⁰.

Providing feedback (Effect size 0.71)

The power of feedback to students learning mathematics was found to have the highest effect when teachers provided feedback data or recommendations to students. Feedback is information given to the learner and/or the teacher about the learner's performance relative to learning goals. It should aim to (and be capable of) producing improvement in students' learning. Feedback redirects or refocuses either the teacher's or the learner's actions to achieve a goal, by aligning effort and activity with an outcome. It can be about the learning activity itself, about the process of activity, about the student's management of their learning or self-regulation or (the least effective) about them as individuals. This instructional feedback can be verbal, written, or can be given through tests or via digital technology and should focus on informing students the specific things they need to do in order to get it right or to improve their performance in some way. It can come from a teacher or someone taking a teaching role, or from peers. In addition, when teachers seek, or at least are open to, feedback from students as to what students know, what they understand, where they make errors, when they have misconceptions, when they are not engaged – the teaching and learning is most powerful. Feedback to teachers makes the learning visible.¹¹

Explicit Teaching Practices (Direct Instruction) (Effect size 0.65)

Explicit teaching practices involve teachers clearly showing students what to do and how to do it, rather than having students discover or construct information for themselves. It recognises that learning is a cumulative and systematic process, starting with building strong foundations in core skills in literacy and numeracy. Effective teacher practices ensure that students have clear instruction on what is expected of them, and what they need to learn from tasks. It ensures that students are given time to engage with the learning process, ask questions and get clear feedback. Students who experience explicit teaching practices make greater learning gains than students who do not experience these practices.

⁶ Gersten et al 2009 cited in <http://www.hanoverresearch.com/media/Best-Practices-in-Math-Interventions.pdf> p8

⁷ ACER p 132

⁸ IES April 2009 What works clearing house Assisting students struggling with mathematics : Response to Intervention for Elementary and Middle Schools p14

⁹ <http://www.ascd.org/publications/researchbrief/v3n13/toc.aspx>

¹⁰ Hattie 2009 p42

¹¹ Hattie p 173

Peer assisted learning (Effect size: 0.62)

The overall effects of the use of peers as co-teachers (of themselves and of others) in classes is, overall, quite powerful. If the aim is to teach students self-regulation and control over their own learning then they must move from being students to being teachers of themselves. One way to achieve this is to use peer tutoring. Peer tutoring in mathematics is effective, when used as a supplement to direct instruction. Cross age tutors (Effect size 0.79) are more effective than same-aged peers (Effect size 0.52) and adult tutors (Effect size 0.54). When students become the teachers of others, they learn as much as those they are teaching. When they have some control or autonomy over this teaching, the effects are higher¹².

Problem solving teaching (Effect size 0.60)

Hattie found significant direct links between problem solving and various measures of basic performance, in particular skills in basic mathematics. A format consisting of full problem statements supported by diagrams, figures or sketches directly related to better performance. His meta-analysis also supported the power of teaching the heuristic method of problem solving, which includes understanding the problem, obtaining a plan of the solution, carry out the plan and examine to solution obtained¹³.

Mathematics and mindset

People with a fixed mindset see intelligence as static which leads to a desire to look smart and therefore a tendency to avoid challenges, give up easily, see effort as fruitless, ignore useful negative feedback and feel threatened. Those with a growth mindset believe intelligence can be developed and leads to a desire to learn and therefore a tendency to embrace challenges, persist in the face of setbacks, see effort as the path to mastery, learn from criticism and find lessons and inspiration in the success of others. As a result, they reach an even higher level of achievement. According to Boaler¹⁴ (2016), the findings that the brain can grow, adapt and change assures us that with the right teaching and messages, children can be successful in Maths, and everyone can achieve at the highest levels of school. Scientific evidence suggests that the ability to be successful comes from the child's approach to life and learning, the messages they receive about their potential and the opportunities they have to learn. "Teacher beliefs and the choices teachers make, can have a major impact on how students view mathematics and their learning of it."¹⁵ Children need to have the self-belief that leads to a mathematical mindset which requires a change in the way students consider themselves and the way they approach mathematics. With a growth mindset (and supported by teachers who appreciate the importance of mathematical mindsets and developing the perspective and strategies to change students' mindsets), student's learning approaches can become more positive and successful.¹⁶

Creating powerful Mathematical Learners

Powerful learners never give up, are persistent, don't need an easy solution, are open to challenges, committed to what they are learning, open to failure, engaged, empowered, celebrate failure, ask the right questions, fail often to succeed sooner, apply their learning and work with information so they can understand it. In order to do this we need to change the conceptual narrative of teachers, from TELL to ASK.

¹² Hattie 2009 p186-7

¹³ Hattie 2009 p210

¹⁴ Boaler J 2016 Mathematical Mindsets USA Jossey-Bass

¹⁵ Principal as Mathematics Leader p11

¹⁶ Blackwell et al 2007 cited in Boaler p6)

Evidence-based approaches in teaching of Mathematics

Children's acquisition of numeracy skills is highly dependent on the effectiveness of classroom teaching. According to The Mathematics Matters Project¹⁷ teaching of mathematics is more effective when it:

Instructional strategy	Further information
<i>Builds on the knowledge learners already have</i>	This means developing formative assessment techniques and adapting our teaching to accommodate individual learning needs.
<i>Exposes and discusses common misconceptions and other surprising phenomena</i>	Learning activities should expose current thinking, create misconceptions and other 'tensions' by confronting learners with inconsistencies and surprises, and allow opportunities for resolution through discussion.
<i>Uses higher-order questions</i>	Questioning is more effective when it promotes explanation, application and synthesis rather than mere recall
<i>Makes appropriate use of whole class interactive teaching and cooperative small group work</i>	Collaborative group work is more effective after learners have been given an opportunity for individual reflection. Activities are more effective when they encourage critical, constructive discussion, rather than argumentation or uncritical acceptance. Shared goals and group accountability are important. Groups should be no more than 6 ¹⁸ . This also allows for students to think aloud as they solve problems.
<i>Encourages reasoning rather than 'answer getting'</i>	Often, learners are more concerned with what they have 'done' than with what they have learned. It is better to aim for depth than for superficial 'coverage.' Encourage use of graphic representation to work through problem-solving options supports reasoning.
<i>Uses rich, collaborative tasks</i>	The tasks used should be accessible, extendable, encourage decision-making, promote discussion, encourage creativity, encourage 'what if' and what if not' questions.
<i>Creates connections between topics both within and beyond mathematics and with the real world</i>	Learners often find it difficult to generalise and transfer their mathematics learning to other topics and contexts. Related concepts and with the real world (such as division, fraction and ratio) remain unconnected. Effective teachers build bridges between ideas.
<i>Uses resources, including technology, in creative and appropriate ways</i>	ICT offers new ways to engage with mathematics. At its best it is dynamic and visual: relationships become more tangible. ICT can provide feedback on actions and enhance interactivity and learner autonomy. Through its connectivity, ICT offers the means to access and share resources and – even more powerfully – the means by which learners can share their ideas within and across classrooms
<i>Confronts difficulties rather than seeks to avoid or pre-empt them</i>	Effective teaching challenges learners and has high expectations of them. It does not seek to 'smooth the path' but creates realistic obstacles to be overcome.... Confidence, persistence and learning are not attained through repeating successes, but by struggling with difficulties
<i>Develops mathematical language through communicative activities</i>	Mathematics is a language that enables us to describe and model situations, think logically, frame and sustain arguments and communicate ideas with precision. Learners do not know mathematics until they can 'speak' it. Effective teaching therefore focuses on the communicative aspects of mathematics by developing oral and written mathematical language
<i>Recognises both what has been learned and also how it has been learned</i>	What is to be learned cannot always be stated prior to the learning experience. After a learning event, however, it is important to reflect on the learning that has taken place, making this as explicit and memorable as possible. Effective teachers will also reflect on the ways in which learning has taken place, so that learners develop their own capacity to learn

¹⁷ ACER Literacy and Numeracy Early years http://research.acer.edu.au/cgi/viewcontent.cgi?article=1019&context=policy_analysis_misc

¹⁸ <http://www.hanoverresearch.com/media/Best-Practices-in-Math-Interventions.pdf>

DECD Leading Numeracy Improvement - 3 effective research based math practice models

This is a synthesis of models from:

- The National Council of Teachers of Mathematics (NCTM) is a highly regarded association for mathematics teaching
- Professor Peter Sullivan, Monash University for Science and Mathematics, the lead writer of the Australian Curriculum Mathematics has extensive experience in research and teacher education.
- Professor Doug Clarke from the Australian Catholic University directs the mathematics teaching research centre and jointly coordinated the National Mathematics Curriculum and Teaching program MCTP which was the flagship for contextual inquiry based learning in mathematics.

The **NCTM Principles to Actions** represent a core set of high leverage practices and essential teaching skills necessary to promote deep learning of mathematics.

Principles to Action Ensuring Mathematical Success for All (NCTM 2014)
Establish Mathematics goals to focus learning Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analysing and comparing student approaches and arguments.
Pose purposeful questions Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Professor Peter Sullivan developed **6 principles for effective teaching of mathematics** synthesised from many studies and research into effective teaching practice in mathematics.

Six Key Principles For Effective Teaching Of Mathematics (Peter Sullivan, 2011)
Articulating goals Identify key ideas that underpin the concepts you are seeking to teach, communicate to students that these are the goals of the teaching, and explain to them how you hope they will learn.
Making connections Build on what students know, mathematically and experientially, including creating and connecting students with stories that both contextualise and establish a rationale for the learning.
Fostering engagement Engage students by utilising a variety of rich and challenging tasks that allow students time and opportunities to make decisions, and which use a variety of forms of representation.
Differentiating challenges Interact with students while they engage in the experiences, encourage students to interact with each other, including asking and answering questions, and specifically plan to support students who need it and challenge those who are ready.
Structuring lessons Adopt pedagogies that foster communication and both individual and group responsibilities, use students' reports to the class as learning opportunities, with teacher summaries of key mathematical ideas.
Promoting fluency and transfer Fluency is important, and it can be developed in two ways: by short everyday practice of mental processes; and by practice, reinforcement and prompting transfer of learnt skills.

25 Characteristics of Effective Maths Teaching (Adapted from Clarke and Clarke)

25 Characteristics of Effective Maths Teaching (Adapted from Clarke and Clarke, 2004)
Mathematical focus 1. Focus on important mathematical ideas 2. Make the mathematical focus clear to the children
Purpose and choice of task 3. Structure purposeful tasks that enable different possibilities, strategies and products to emerge 4. Choose tasks that engage children and maintain involvement
Materials, tools and representations 5. Use a range of materials/representations/contexts for the same concepts
Adaptations, connections and links 6. Use teachable moments as they occur 7. Make connections to mathematical ideas from previous lessons or experiences
Organisational style(s), teaching approaches 8. Engage and focus children's mathematical thinking through an introductory whole group activity 9. Choose from a variety of individual and group structures and teacher roles within the major part of the lesson
Learning community and classroom interaction 10. Use a range of question types to probe and challenge children's thinking and reasoning 11. Hold back from telling children everything 12. Encourage children to explain their mathematical thinking/ideas 13. Encourage children to listen and evaluate others' mathematical thinking/ideas, and help with methods of understanding 14. Listen attentively to individual children 15. Build on children's mathematical ideas and strategies
Expectations 16. Have high but realistic mathematical expectations of all children 17. Promote and value effort, persistence and concentration
Reflection 18. Draw out key mathematical ideas during/and/or towards the end of the lesson 19. After the lesson, reflect on children's responses and learning, together with activities and lesson content
Assessment methods 20. Collect data by observation and/or listening to children, taking notes as appropriate 21. Use a variety of assessment methods 22. Modify planning as a result of assessment
Personal attributes of the teacher 23. Believe that mathematics learning can and should be enjoyable 24. Are confident in their own knowledge of mathematics at the level they are teaching 25. Show pride and pleasure in individuals' success

DECD Leading Numeracy Improvement– 8 effective practices that develop Numeracy B-18

<p style="text-align: center;">Fostering engagement</p> <p>Inspire learners through a visible passion for mathematics learning. Engage learners in rich and challenging tasks that allow time and opportunities to make decisions and position mathematics as both a valuable tool for human endeavour and as a source of interest and wonder. (TfEL 3.3, 4.2) (RRR A.L.S, I.S) (EYLF—Practices 1, 2, 3, 4,5)</p> <p>Whole Site: Across our site how do our learning programs provide a variety of rich and challenging experiences to foster engagement across all groups of learners?</p> <p>Educator: How do you model your own engagement and enthusiasm for thinking and learning mathematically?</p> <p>Learner: How do you hook yourself into your learning?</p>	<p style="text-align: center;">Making connections</p> <p>Build on what learners know, mathematically and experientially, in ways that both contextualise and establish a rationale for learning. Support learners to connect mathematics learning to their lives in local, global and broader contexts. Multiple representations of mathematical concepts are used to deepen understanding. (TfEL 4.2, 4.4) (RRR A.L.S, W.S, I.S, R.S) (EYLF—Practices 1, 2, 3, 5)</p> <p>Whole site: What examples do we have that we design experiences to build on prior skills and knowledge?</p> <p>Educator: In what ways are learners supported to use multiple representations and connect their learning in broader contexts?</p> <p>Learner: Are your learning experiences meaningful to your everyday life?</p>	<p style="text-align: center;">Collecting and responding to evidence</p> <p>Use evidence of learner’s thinking to assess their progress toward mathematical understandings in a range of contexts. Analyse this evidence to inform feedback, instruction and future planning. (TfEL 4.1, 4.3) (RRR W.S, I.S) (EYLF—Practices 1, 4, 8)</p> <p>Whole site: What are the multiple ways that we know where each learner is going, how they are going and where they need to go to next?</p> <p>Educators: What evidence of learner thinking are you using and how are you using it to inform your planning?</p> <p>Learners: How does the feedback you receive help you to move forward?</p>
<p style="text-align: center;">Identifying learning goals</p> <p>With learners, establish clear mathematics and numeracy learning goals and intentions that reflect their individual learning needs and develop positive dispositions. (TfEL 2.1, 2.3, 4.1) (RRR R.S, W.S, I.S) (EYLF—Practices 2, 4, 5, 6, 8)</p> <p>Whole site: In what ways do we develop and communicate our learning intentions so that everyone is clear on where they are going?</p> <p>Educator: What processes do you have to work with learners to set goals and targets and review their progress as a continual process?</p> <p>Learner: Are you involved in setting your learning goals and working towards your targets and challenges?</p>	<ol style="list-style-type: none"> 1. How often do learners experience these practices and how do you know? 2. Which of these practices are strengths and which ones need to be developed further? 	<p style="text-align: center;">Building fluency from conceptual understanding</p> <p>Build fluency with procedures on a foundation of conceptual understanding so that over time learners become skilful in using procedures flexibly as they solve contextual and mathematical problems. (TfEL 3.2) (RRR R.S, A.L.S, W.S, I.S) (EYLF—Practices 2, 5, 8)</p> <p>Whole site: Do we recognise the value of exploring the concept and meaning making as the first part of the learning?</p> <p>Educators: How do you provide a balance between learners exploring a concept and educational direction?</p> <p>Learners: Are you provided with time, space and resources to direct your learning?</p>
<p style="text-align: center;">Facilitating meaningful collaboration and dialogue</p> <p>Create a classroom community where mathematical dialogue is critical to the development of shared understanding. This occurs through collaborative learning tasks that require learners to demonstrate their ability to confidently share their mathematical reasoning and to critique and build on the reasoning of others. (TfEL 2.2, 3.1, 3.4) (RRR A.L.S, W.S, I.S) (EYLF—Practices 1, 2, 3, 5)</p> <p>Whole site: What are the multiples ways that we facilitate meaningful, collaborative dialogue?</p> <p>Educator: <i>How do you facilitate experiences that provide opportunities for purposeful dialogue and collaborative learning?</i> <i>How do you invite student dialogue through purposeful questioning?</i></p> <p>Learner: In what ways do you share your thinking and reasoning with others?</p>	<p style="text-align: center;">Providing challenge</p> <p>Engage learners in challenging mathematics tasks with multiple entry and exit points. Provide opportunities and time for students to engage in ‘productive struggle’, developing persistence, confidence and problem-solving skills. (TfEL 2.4, 3.2) (RRR A.L.S, I.S) (EYLF— Practices 1, 2, 3, 4, 5)</p> <p>Whole site: How effectively does our design of learning encourage all learners to engage their higher order thinking and grapple with problem solving?</p> <p>Educator: How do you work actively to build and maintain a positive and challenging environment for all learners?</p> <p>Learners: What do you do when the learning is challenging?</p>	<p style="text-align: center;">Using digital technology to connect</p> <p>Engage learners with digital technologies as essential resources that support their learning and enable them to connect with real contexts with real purpose, reason mathematically and communicate their thinking. (TfEL 4.2, 4.4) (RRR A.L.E) (EYLF— Practices 2, 3, 4, 5)</p> <p>Whole site: What opportunities are there for learners to investigate, communicate and create using digital technology?</p> <p>Educators: How do you know that digital technology is used purposefully?</p> <p>Learners: How have you used digital technology in your learning recently?</p>

The 'Big' Ideas in Number¹⁹

The Big Ideas in Number is the work of Dianne Siemon, Professor of Mathematics at the School of Education at RMIT University (Victoria) and arose from the Scaffolding Numeracy in the Middle Years (SNMY) Research Project and the thinking behind the Assessment for Common Misunderstanding resources. A big idea is a strategy, or way of thinking about some key aspects of mathematics, without which students' progress in mathematics will be seriously impacted.

Year	Definition	Information
End Reception	Big ideas identified for assessment of common misunderstandings tools	The tools are based on a series of highly focussed, research-based Probe Tasks and comprises a number of easy to administer, practical assessment tasks designed to address a key area of Number across 6 levels (equates to Foundation- end Year 10).
End Reception	Trusting the count - developing flexible mental objects for numbers 0-10	Trusting the count has two meanings: <ul style="list-style-type: none"> Initially, children may not believe that if they counted the same collection again, they would get the same result, or that counting is a strategy to determine how many (WA First Steps in Mathematics Project, 2004). Ultimately, it is about having access to a range of mental objects for each of the numbers to ten, which can be used flexibly without having to make, count or see these collections physically
End Yr 2	Place value - importance of moving beyond counting by ones, the structure of the base 10 numeration system	A system of assigning values to digits based on their position (e.g., in a base ten system of numeration, positions represent successive powers of ten). At all levels this requires: <ul style="list-style-type: none"> modelling and/or representing numbers in appropriate ways (manipulatives, number lines, diagrams, etc); naming (hear, say, read, and write in words); recording (write in numerals); and consolidating by comparing, ordering, counting forwards & backwards in place-value parts, & renaming numbers in multiple ways
End Yr 4	Multiplicative thinking - the key to understanding rational number and developing efficient written and mental computation strategies in later years	Capacity to work with an extended range of concepts, meanings, and representations for multiplication and division in a variety of contexts. Multiplicative thinking has been characterised by: <ul style="list-style-type: none"> A capacity to work flexibly and efficiently with an extended range of numbers (e.g., larger whole numbers, decimals, common fractions, ratio, & percent); An ability to recognise & solve a range of problems involving multiplication &/or division including direct and indirect proportion; The means to communicate this effectively in a variety of ways (e.g words, diagrams, symbols, written algorithms).
End Yr 6	Partitioning - the missing link in building common fractional and decimal knowledge and confidence	Partitioning/equipartitioning: the process of dividing (usually physically) a quantity or collection into equal parts with no remainder. Partitioning is the key to formalising and extending fraction ideas. By developing strategies for thirding & fifthing based on halving, students can be supported to: <ul style="list-style-type: none"> notice key generalisations; create fraction diagrams and number line models; make connections to the region, for each, and factor ideas for multiplication; and make, name, compare, and rename mixed and proper fractions
End Yr 8	Proportional reasoning: extending what is known about multiplication & division beyond rule based procedures to solve problems involving fractions, decimals, percent, ratio, rate & proportion	

¹⁹ <https://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/devbigideas.pdf> and <http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/misunderstandings.aspx>

Evidence Based Programs

Please note that all recommendations are taken from ACER Literacy and Numeracy Interventions in the Early Years of Schooling unless otherwise footnoted

Australian Origin				Wave 1	
Program	Origin	Target gp	Yr	Intervention focus	Recommendation
Count Me In Too (CMIT)	NSW	All students	K-6	Trough PD, teachers develop a better understanding of how children learn arithmetic. They then develop teaching strategies based on their understanding of each child's development in terms of the Literacy Framework in Number (LFIN)	Some reliable evidence²⁰
Count me In Too Indigenous	NSW	ATSI students	K-2	The principles of CMIT were adapted to be culturally appropriate for Aboriginal children	Very limited evidence available
First steps in Mathematics	WA	All students	F-2	PD focuses on enhancing teacher knowledge in the 4 strands of Mathematics and to better understand children's mathematical difficulties	No available research evidence
Learning in Early Numeracy	NSW	All students	K-4	PD for teachers enhances their understanding of children's development of number concepts in the context of growth points of the Early Numeracy research project (ENRP) so as to better intervene to promote children's understanding	No available research evidence
Mathematics in Indigenous Contexts Project	NSW	ATSI students	K-6	Learning teams in schools comprising teachers, parents and mentors collaborated to develop mathematics units of work contextualised for Aboriginal students.	Very limited evidence of efficacy and effectiveness
Numeracy Matters	NSW	All students	3-6	Online PD for teachers to develop shared understanding of numeracy development is part of a whole school approach to improve the teaching of numeracy	No available research evidence
Scaffolding Numeracy in the Middle Years	Victoria	All students	4-8	Based on an assessment guided approach to improving student numeracy outcomes, in particular identifying and refining a learning and assessment framework for the development of multiplicative thinking at this level using rich assessment tasks. Provides a Learning and Assessment Framework for Multiplicative Thinking , assessment materials for multiplicative thinking , Learning plans , Authentic tasks and a resource library . Available from: http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/scaffoldnum.aspx .	Currently being trialled nationally and 6 SA schools have participated in the research.
Success in Early Numeracy	Victoria	All students	F-6	PD to develop teachers' understanding of children's mathematical development of number concepts using clinical interviews to assess children's understanding and the growth points of the early to describe children's achievement	Insufficient evidence to determine impact
Taking off with Numeracy (also a Wave 2 intervention program)	NSW	All students, + those targeted as having low attainment in Maths	3-6	Teachers undertake PD to improve their understanding of how children develop their mathematical understanding. In class intervention for targeted students is designed to help them develop more efficient strategies and higher-order mathematical thinking.	Limited evidence of impact on student achievement





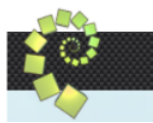
²⁰ ACER 2015 Literacy and Numeracy Interventions in the Early Years of Schooling: A Literature Review p xii

Overseas Origin				Wave 1	
Program	Origin	Target group	Yr	Intervention focus	Recommendation
Building Blocks	USA	All students	K-2	The program uses the framework of learning trajectories for the development of children's mathematical thinking, with activities embedded in the program designed to encourage the development of conceptual thinking in these trajectories.	Limited evidence of impact for pre-school children; no evidence for older students
DreamBox Learning	USA	All students	K-6	Online program focussing on number and operations, place value and number sense	Potentially positive effects with small evidence base ²¹
EnVisionMath	USA	All students	K-6	Classroom computer based program accessible from school and home	Potentially positive effects with small evidence base ²²
Everyday mathematics	USA	All students	K-6	PD for teachers supports the implementation of the program, which focuses on developing students' informal knowledge of mathematics and assisting them to make connections to formal mathematical concepts. Small group work, problem solving, discussion and the use of concrete manipulatives are features of the program.	No evidence of impact for K-2; very limited for Year 3-5 students

²¹ [Hanover Research 2014 Best Practices in Math Interventions](#) p24

²² [Hanover Research 2014 Best Practices in Math Interventions](#) p26

Some Maths websites:

Wuzzit trouble		<p>Helps students gain an understanding of important maths – adding and subtracting, factors and multiples at the same time as they develop number flexibility and problem solving strategies. wuzzittrouble.com/</p>
Mathbreakers		<p>A video game targeted at primary levels – similar to Minecraft but with numbers. Produced by Imaginary Number Co www.mathbreakers.com/</p>
Number Rack		<p>An app targeted at primary levels that models a learning tool called a Rekenrek (0 frames) www.mathlearningcenter.org/web-apps/number-rack</p>
Motion Math		<p>Targeted at primary levels, offers a range of games that help students develop a visual understanding of important maths concepts, particularly fractions and numbers. www.motionmathgames.com</p>
NRich		<p>The nrich site contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world from Kindergarten to Secondary. http://nrich.maths.org</p>

Popular RESOURCES²³:

Brain plasticity

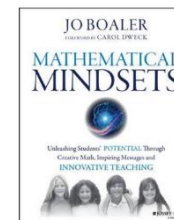
Video clip (5 mins) <https://www.youtube.com/watch?v=pxru8H6XbR4>

Professor Jo Boaler, Stanford University, presents an argument, supported by recent brain research, that challenges the belief that some people are good at maths and some are not.

Mathematical mindsets

Boaler J (2016) *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*, San Francisco: Jossey-Bass

A very readable book that includes chapters about the brain and learning maths; creating mathematical mindsets; rich tasks; the path to equity; and teaching maths for a growth mindset. Her one page handouts of her **seven positive norms** for students (and others) are valuable resources. (also available on youcubed website)



Beliefs and attitudes about mathematics/numeracy

A range of surveys for Pre-school, Primary and Secondary level students, teachers and parents to audit the beliefs and attitudes of your learners, educators and their community Can be found at the Leading Numeracy Improvement Edmodo site (Join code ncuifv)

Handout for parents: youcubed website

<https://www.youcubed.org/handout-for-parents/>

Six tips for parents to support their child's mathematical learning, together with links to other Youcubed parent resources.



The principal as mathematics leader

Ontario Principals' Council (2009) *The principal as mathematics leader*, California: Corwin Press (76 pages + principal resources)

An overview of how school administrators can start supporting mathematics education in their schools, along with advice about observing and evaluating classrooms, actions principals take, tools for success, and resources.



Advice for graduates

'Becoming a professional teacher of mathematics', Chapter 29 in Siemon, D., Beswick, K., Clark, J., Faragher, R., & Warren, E. (2011) *Teaching mathematics: Foundations to middle years*, Melbourne: Oxford University Press

²³ Collated by Pauline Carter DECD Results Plus
Leanne Prior 2016

How to learn maths—for students Jo Boaler, Stanford University (<https://lagunita.stanford.edu/courses/Education/EDUC115-S/Spring2014/about>)

Free Online Class (also appropriate for teachers and parents)

Students learn about their own math potential; strategies to learn and relate well to maths; the brain and math strategies and how to be more powerful in maths classes and in life! NB There is also a more extensive fee-paying professional learning course for teachers and parents.



Make it count maths website <http://mic.aamt.edu.au/>

Make it count is a teaching and learning resource and a professional learning tool for educators working with Aboriginal and Torres Strait Islander learners in mathematics education. It offers pathways, possibilities and ideas for schools and professional learning communities to make their own inroads and innovations into improving mathematics and numeracy learning outcomes for Indigenous learners.

Knowledge Resource Book for teachers

A knowledge resource book helps teacher develop a real understanding of the mathematics they will teach and the most effective methods of teaching math topics. Every teacher should have access to this type of resource when designing learning for conceptual understanding, addressing student misconceptions and providing intellectual stretch in mathematics.

Van De Walle JA, Karp KS & Bay-Williams JM (2014) *Elementary and middle school mathematics. Teaching developmentally*, Harlow: Pearson Education Limited.

This resource guides teachers to help all learners make sense of maths with the emphasis placed on teaching conceptually, in a problem-based, developmentally appropriate manner that supports the learning needs of all students. Includes Pause and Reflect prompts and Activities

Additional Supporting resource: The Van de Walle *Professional Mathematics Series* Vol 1: PreK-2, Vol 2: Yr 3-5 Vol 3: Yr 6-8

Siemon D, Beswick K, Clark J, Faragher R & Warren E (2011) *Teaching mathematics: Foundations to middle years*. Melbourne: Oxford University Press.

Mathematics Conceptual Development

The *Mathematics Developmental Continuum F–10* provides evidence based indicators of progress, linked to powerful teaching strategies, aligned to the progression points.

<http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/continuum/Pages/mathcontin.aspx>

Assessment of Concept Development

The *Scaffolding Numeracy in the Middle Years* (SNMY) is a new assessment-guided approach to improving student numeracy outcomes in Years 4 to 8.

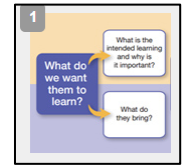
Assessment for Common Misunderstandings - These assessment tools are based on a series of highly focussed, research-based Probe Tasks and the Probe Task Manual also includes a number of additional tasks and resources which have been organised to address common misunderstandings.

<http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/mathsassess.aspx>

Designing Learning:

Teachers need to be clear about their learning intentions. The Learning Design template supports teachers to clarify what they want students to learn, how they will know if they got there and what they will do to get them there.

http://www.acleadersresource.sa.edu.au/index.php?page=learning_design



Teachers must reference the learning to the relevant curriculum documents.

http://www.acleadersresource.sa.edu.au/index.php?page=what_you_value

Learning Area Explorers for every year level F-10 allow teachers to investigate and track connections in the Australian Curriculum eg. Content Descriptions, Proficiencies, Achievement Standards, Year Level Descriptions.

General Capabilities Continua provide benchmark developmental levels for all GC including Early Literacy, Literacy, Numeracy and Critical and Creative Thinking.

Effective Questioning

Effective questioning is an important skill for teachers to scaffold conceptual understanding, identify and address misconceptions and assess learning.

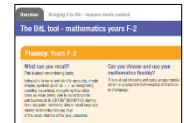
Bringing it to Life Tool has year level appropriate questions for Fluency, Understanding, Reasoning and Problem Solving.

(*BitL Printables* http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life)

Asking Effective Questions

A very practical and comprehensive reading with the why, the how and the what of effective questioning.

http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/CBS_AskingEffectiveQuestions.pdf



Rich Learning Experiences

Learning experiences need to be designed to differentiate challenge for every learner with multiple entry and exit points (low floor/high ceiling). Always consider the level of student thinking required.

Transforming Tasks: A resource to support a pedagogical shift from teacher instructed to learner constructed where students are doing the thinking, using 4 strategies:

From Tell to Ask, From Closed to Open, From Procedural to Problem Solving, From Information to Understanding.

http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom Preschool and Year 12 Maths example on USB

Conceptual Narratives: This resource is not yet fully developed. These resources exemplify learning experiences that bring the Proficiencies and content together, supporting learners to construct their own knowledge.

Printables: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life



Electronic Resources:



Dan Meyer's blog: 101 questions: <http://www.101qs.com> Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*
Also: Dan Meyer 3 Act Lessons.

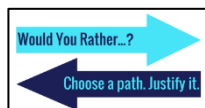


Estimation 180 is a website with a bank of daily estimation challenges to help students to improve both their number sense and problem solving skills.
<http://www.estimated180.com/>



This **Illuminations** website provides access to quality resources for teaching and learning mathematics, including interactive tools for students and instructional support for teachers. Pre-K -12

<https://illuminations.nctm.org/>



Would you rather? A website with multiple situations where students are asked to think deeply about two options, choose and then justify their choice.
<http://www.wouldyourathermath.com/> (Also Which One Doesn't Belong? <http://wodb.ca/>)

A week of Inspirational Maths - You cubed Website



These resources intend to provide important growth mindset messages that will help students feel confident, try harder all year, persist with open and difficult problems and embrace mistakes and challenge. All tasks are low floor and high ceiling – they are accessible to all students and they extend to high levels. <https://www.youcubed.org/week-of-inspirational-math/>



Thought Provoking Maths resource provides examples of a model for designing mathematical activities with provocations, investigations, experimentation, problem solving and skill development and represent learning. This is a simple entry point and in conjunction with Learning Design can enrich mathematical learning.

Reading

Westwell, Val, 2015. AAMT Biennial Conference paper: **Leading Learning. Going beyond the content.** http://www.acleadersresource.sa.edu.au/features/transforming-tasks/AAMT_PAPER_Val_Westwell_July_2015.pdf

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Early Numeracy Research Project :<http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/Pages/enrp.aspx>

Hanover Research 2014 *Best Practices in Maths Interventions* <http://www.hanoverresearch.com/media/Best-Practices-in-Math-Interventions.pdf>

Hattie 2009 *Visible Learning* London & New York Routledge

Hattie *Interactive Visualisation of Effect Sizes* <http://visible-learning.org/hattie-ranking-influences-effect-sizes-learning-achievement/>

<http://www.educatorstechnology.com/2014/01/20-great-math-websites-for-teachers-and.html>

<https://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/devbigideas.pdf>

<http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/misunderstandings.aspx>

IES April 2009 *Assisting students struggling with mathematics : Response to Intervention for Elementary and Middle Schools* What Works Clearing House

MUSEC Briefing Issue 39 July 2014 *What is Direct Instruction?* https://www.mq.edu.au/_data/assets/pdf_file/0019/171037/MUSEC-Briefing-39-What-is-direct-instruction.pdf

Ontario Principals Council 2009 *Principal as Mathematics Leader* Ontario Principals' Council

Teaching and Learning Toolkit Australia <http://evidenceforlearning.org.au/the-toolkit/>

Using data from Mathematics Assessment Interview: <https://www.acer.edu.au/files/eppc15-Bianca-Witney.pdf>

Wendling B & Mather N *Essentials of Evidence-Based Academic Interventions* 2009 New Jersey John Wiley & Sons

What Works Clearing House: [http://ies.ed.gov/ncee/wWc/FindWhatWorks.aspx?o=1,9,13&n=Mathematics achievement](http://ies.ed.gov/ncee/wWc/FindWhatWorks.aspx?o=1,9,13&n=Mathematics%20achievement)