Evidence based Mathematics Interventions (Wave 2 and 3)

Effective literacy and numeracy interventions are embedded in whole-school approaches to improving literacy and numeracy for all students. Early diagnosis and intervention is vital for students at risk of not progressing numeracy learning[1] to eliminate future struggles with increasingly complex and abstract mathematical concepts[2].

Recommendations and levels of evidence[3]:

Recommendation	Evidence Level
Wave 1	
Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk	***Moderate***
Waves 2 and 3	
Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalisation of thought processes, guided practice, corrective feedback, and frequent cumulative review	***Strong***
Interventions should include instruction on solving word problems that is based on common underlying structures	***Strong***
Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts	***Moderate***
Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas	***Moderate***
Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in Preschool through Year 5 and on rational numbers in Years 4 through 8. These materials should be selected by committee	Low
Monitor the progress of students receiving supplemental instruction and other students who are at risk	Low
Include motivational strategies in Wave 2 and Wave 3 interventions	Low

Recommendations and levels of evidence for Wave 2 and 3 Interventions:

INSTRUCTIONAL STRATEGY	EFFECT SIZE **Indicates a large or moderate to large effect size*	
	SPECIAL EDUCATION STUDENTS	UNDER -ACHIEVING STUDENTS
Systematic and explicit instruction	***1.19****	***0.58****
Student think alouds	0.98*	N/A
Visual and graphic descriptions of problems	0.50	N/A
Use of structured peer-assisted learning activities using heterogeneous ability grouping	0.42	0.62*
Formative assessment data provided to teachers	0.32	0.51
Formative assessment data provided directly to students	0.33	0.57*
Include Schema-based instruction which explicitly teaches strategies to translate text into semantic representation of a problem (eg story, diagram)	Recommended[4]	Recommended

[1] ACER http://research.acer.edu.au/cgi/viewcontent.cgi?article=1019&context=policy_analysis_misc p131

[2] http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf p6

[3] [3] Gersten et al 2009 cited in http://www.hanoverresearch.com/media/Best-Practices-in-Math-Interventions.pdf p8

[4] Schema Based Instruction for Mathematics Word Problem-solving Recommended by MUSEC Briefing Issue 27 May 2011

Further information: Evidence-based Strategies

What instructional strategies are effective in helping students with difficulties in mathematics[5]?

Research points to several strategies that have been consistently effective in teaching students who experience difficulties in mathematics.

- The use of structured peer-assisted learning activities
- Systematic and explicit instruction using visual representations
- Modifying instruction based on data from formative assessment of students (such as classroom discussions or quizzes)
- Providing opportunities for students to think aloud while they work

What are the characteristics of students with learning difficulties in Mathematics?

Students who struggle with mathematics learning regardless of their motivation, past instruction and mathematical knowledge prior to starting school;

- Demonstrate slow or inaccurate recall of basic arithmetic facts
- Answer problems impulsively, without inhibition
- Have difficulty representing mathematical concepts mentally
- Have poorly developed number sense; and
- Have difficulty keeping information in the working memory

Screen all students to identify those at risk

Screening measures that are efficient and reasonably reliable and that demonstrate predictive validity should be used. It is recommended that screening should occur in the beginning and middle of the year and be based on the content and instructional objectives for each year level[6]. This should be correlated with Naplan and PAT-M data.

Instruction during the intervention should be explicit and systematic.

Explicit and systematic instruction for students at the Wave 2 and 3 level has the strong evidence-based as best practice for providing mathematics interventions. It is recommended that struggling students receive some explicit instruction regularly and that some of the explicit instruction ensure that students possess the foundational skills and conceptual knowledge necessary for understanding their year-level mathematics[7].

Explicit instruction for students experiencing difficulties is defined as[8]:

- teachers providing clear models for solving a problem type using an array of examples and demonstrated a step-by-step plan (strategy) for solving the problem;
- students receiving extensive practice in use of newly learned strategies and skills
- students actively encouraged to use the same procedure/steps demonstrated by the teacher
- students provided with opportunities to think aloud (i.e., talk through the decisions they make and the steps they take

[5] http://www.nctm.org/uploadedFiles/Research_and_Advocacy/research_brief_and_clips/Student_with_Difficulties_Clip.pdf
[6] http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf p13-15
[7] http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf p21
[8] Gersten et al 2009 cited in http://research.acer.edu.au/cgi/viewcontent.cgi?article=1019&context=policy_analysis_misc p 120 and http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf p 21

- students provided with extensive feedback

However, there is no evidence to support explicit instruction as the ***only*** effective teaching approach. There is further evidence for:[9]

- the use of visual examples
- careful attention to the sequencing and selection of examples taught in numeracy interventions
- encouraging students to verbalise their thinking in solving problems
- provided models of proficient problem solving
- guided practice
- corrective feedback
- providing feedback to teachers about the progress of intervention students

Interventions should include instruction on solving word problems that is based on common underlying structures

Students who have difficulties in mathematics typically experience severe difficulties in solving word problems related to the mathematics concepts and operations they are learning. This is a major impediment for future success in any math-related discipline. It is recommended that interventions include systematic explicit instruction on solving word problems, using the problems' underlying structure. Simple word problems give meaning to mathematical operations such as subtraction or multiplication. When students are taught the underlying structure of a word problem, they not only have greater success in problem solving but can also gain insight into the deeper mathematical ideas in word problems. Systematic instruction should be given on the structural connections between known, familiar word problems and unfamiliar, new problems. By making explicit the underlying structural connections between familiar and unfamiliar problems, students will know when to apply the solution methods they have learned[10].

Intervention materials should include opportunities for students to work with visual representations of mathematical ideas

A major problem for students who struggle with mathematics is weak understanding of the relationships between the abstract symbols of mathematics and the various visual representations. Student understanding of these relationships can be strengthened through the use of visual representations of mathematical concepts such as solving equations, fraction equivalence, and the commutative property of addition and multiplication.

Such representations may include:

- number lines
- graphs
- simple drawings of concrete objects
- simplified drawings

The ability to express mathematical ideas using visual representations and to convert visual representations into symbols is critical for success in mathematics. A major goal of interventions should be to systematically teach students how to develop visual representations and how to transition these representations to standard symbolic representations used in problem solving. Occasional and unsystematic exposure (the norm in many classrooms) is insufficient and does not facilitate understanding of the relationship between the abstract symbols of mathematics and various visual representations[11].

[9] ACER http://research.acer.edu.au/cgi/viewcontent.cgi?article=1019&context=policy_analysis_misc p 121 and http://www.hanoverresearch.com/media/Best-Practices-in-Math-Interventions.pdf
[10] http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf pp26-29
[11] http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf pp30-35

Interventions should build number sense and fluent retrieval of basic arithmetic facts

The strong relationship between early number sense measures and later mathematical performance are compelling. Quick retrieval of basic arithmetic facts is critical for success in mathematics. Yet research has found that many students with difficulties in mathematics are not fluent in such facts. Weak ability to retrieve arithmetic facts is likely to impede understanding of concepts that students encounter with rational numbers[12].

Evidence highlights the importance of **early assessment** in the diagnosis of children's understandings (eg by using Assessment for common misunderstandings[13] et al) rather than waiting to intervene should difficulties be apparent. Interventions should ideally occur in Year 1 or 2.[14] One indicator which appears to effectively discriminate children who experience mathematical difficulties from those who are able to benefit from classroom instruction is the use of efficient and increasingly sophisticated counting strategies[15].

Central to children's development of mathematical thinking in the early years is **number sense** which includes the capacity to identify small numbers, reason about larger and smaller numbers, and the results of simple transformations (e.g. adding and subtracting one). There is evidence to believe that children who commence school with poorly developed number sense, are less equipped to take advantage of classroom instruction[16]. Children with a well-developed number sense in Preschool have a good procedural grasp of the counting sequence, but more importantly children with number sense understand the uses of counting to work out how many and compare different numbers of objects.

In summary, number sense includes:

- fluency in estimating and judging magnitude
- ability to recognise unreasonable result
- flexibility when mentally computing
- ability to move among different representations and to use the most appropriate representation for a given situation and
- ability to represent the same number or function in multiple ways, depending on the context or purpose of this representation

For students in **Reception - Year 1,** it is recommended to explicitly and systematically teach strategies for efficient counting to improve the retrieval of mathematics facts as a step toward automatic, fluent retrieval (eg counting-up strategy for addition). There is evidence that strategy based instruction for fact fluency (such as teaching the counting-on procedure) is superior to rote memorisation.[17]

It is recommended to provide 5-10 minutes each intervention session for practice to help students become automatic in retrieving basic arithmetic facts, **beginning in Year 2**. The goal is quick retrieval of facts using the digits 0 to 9 without any access to pencil and paper or manipulatives. Suggestions include using technology, flash cards, and other materials for extensive practice to facilitate automatic retrieval.[18] It is also recommended to integrate previously learned facts into practice activities so that retrieval becomes automatic

[12] http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf pp37-40
[13] http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/misunderstandings.aspx
[14] ACER p 122
[15] ACER p 123
[16] ACER p123
[17] http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf p38
[18] http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf p38

Evidence Based Programs Waves 2 & 3

Overall there is a lack of high quality research evidence for the efficacy of numeracy interventions implemented in Australia or internationally.[19]
(Please note that all recommendations are taken from ACER Literacy and Numeracy Interventions in the Early Years of Schooling unless otherwise footnoted)

Australian Origin					
Program	**Origin**	**Target group**	**Yr**	**Intervention focus**	**Evidence**
Best Start Targeted Early Numeracy (TEN)[20]	NSW	Students with low attainment in Maths	K-2	PD provided to *TEN* facilitators who in turn provide professional learning to teachers to improve their understanding of mathematical development, and to support teachers to identify and address the learning needs of targeted students.	Limited evidence of efficacy; warrants further investigation
Extending Mathematical Understanding Intervention	***Vic***	***Students with low attainment in Maths***	***1-2***	***PD for specialist teachers to improve their understanding of mathematical development, equip them to administer the 1-1 interview and describe children's understanding in relation to growth points.***	***Some research evidence***
Getting ready in Numeracy (GRIN)	Vic	Students with low attainment in Maths	3	GRIN tutors withdraw students in small groups and undertake short sessions which familiarize them with the context of their upcoming mathematics lesson.	Insufficient evidence to determine impact
Mathematics Intervention	Vic	Students with low attainment in Maths	1-4	Teachers undertake PD to gain familiarity with the clinical interview method. Teachers work with small groups of students using tasks designed to progress children's understanding on Steffe and colleagues counting stages[21].	No available research evidence
Mathematics Recovery	NSW	Students with low attainment in Maths	1	Teachers trained in Mathematics recovery methods assess children, describe their current number knowledge using the SEAL and work with them Individually and intensively using targeted teaching strategies designed to progress their understanding	Limited evidence of the impact on student achievement
Maths for Inclusion (M4LI)	SA	Recommended for students who need intervention in number concepts	***3+***	Part 1: Tasks to move students on in: numbers 1-10 (Structuring Number); Numbers 1-20 (Addition & Subtraction); Using the Base 10 system Part 2: Tasks to move students on in: Saying, reading and Writing Numerals; Beginning Multiplication & Division; multiplication & Division	No evidence found
Numeracy intervention Project	NSW	Students with low attainment in Maths	1,4, 8	Teachers undertake PD to increase their understanding of mathematical development and gain familiarity with the Schedule for Early Number Assessment - SENA. They work individually and intensively with students selected for intervention.	Very limited evidence for the efficacy on student achievement
Numeracy Intervention Research Project	Vic	Students with low attainment in Maths, low SES schools	3-4	Teachers work intensively with individual students or with small groups, with the aim of developing effective instructional approaches to facilitating number development among low attaining students.	No available research evidence

[19] ACER p 116
[20] http://www.scootle.edu.au/ec/viewing/S7059/pdf/tls48_targeted_early_numeracy.pdf
[21] https://books.google.com.au/books?id=tK_xJbSyQMIC&pg=PA9&lpg=PA9&dq=Steffe+and+colleagues+counting+stages&source=bl&ots=CWhqsYl2ul&sig=5mv3E7eq-HelAp7BJmMj720Eqxo&hl=en&sa=X&ved=0ahUKEwj_4PyF6tjOAhUJlJQKHQtsCEAQ6AEIHTAA#v=onepage&q=Steffe%20and%20colleagues%20counting%20stages&f=false

Quicksmart Numeracy	***NSW***	***Students with low attainment in Maths***	***5-8***	***Students are withdrawn from class in pairs and undertake an intensive intervention designed to improve automaticity and speed retrieval for basic arithmetic facts.***	***Wide range of evidence[22]; Moderate evidence[23]***
Program	**Origin**	**Target group**	**Yr**	**Intervention focus**	**Evidence**
Taking off with Numeracy (also a Wave 1 intervention program)	NSW	All students, students targeted as having low attainment in Maths	3-6	Teachers undertake PD to improve their understanding of how children develop their mathematical understanding. In class intervention for targeted students is designed to help them develop more efficient strategies and higher-order mathematical thinking.	Limited evidence of impact on student achievement
Too Smart	SA	Student not reaching potential	1-2	A targeted intervention program developed by Seacliff Primary School	No independent evidence found
Train a Maths Tutor Program	QLD	ATSI students	3-10	Focus on training IEW's (SSOs) to better understand mathematical concepts so as to better assist ASTI children in the classroom.	No evidence on student achievement

Overseas Origin					
Program	**Origin**	**Target group**	**Yr**	**Intervention focus**	**Evidence**
Fraction Face-Off!	USA	Students with low attainment in Maths	4	Explicit instruction in small group tutoring program to address: the part/whole interpretation of fractions and the measurement interpretation of fractions, using an athletics theme	Effect size 0.92[24] on student's ability to compare and calculate fractions
Hot Math Tutoring	***USA***	Students with low attainment in Maths	3	Designed to improve mastery of word-problems in small groups for 20-30 mins 3x week for 13 weeks. Can be implemented alongside the *Classroom Hot Math* program	0.60 Effect size[25]
Number Worlds	USA	Students who are one or two years below year-level	Pre K-8	Teacher-led math intervention intensive targeted program to bring math-challenged PreK-8 students up to grade level with Real World Applications. Pre K-1: prevention; Yr 1-8: Intervention; Yr 6-8: algebra readiness	No reliable evidence available
Numeracy Recovery	UK	Students with low attainment in Maths	2	Teachers undertake PD to implement the intervention, which involves identifying specific areas of need in 10 components of early numeracy and developing individualized instruction to promote learning in these areas.	Limited evidence to assess efficacy
Number Rockets	***USA***	***Students with low attainment in Maths***	***1***	***Teachers undertake PD focused on implementing the initiative. Students receive additional mathematics instruction in small groups with content focused on the development of number sense.***	***Some evidence of effectiveness***

[22] The quality of the research evidence for QSN varies widely and there is a need to explore further the claims that the narrow focus of the intervention on automaticity promotes broader improvements in Maths. http://research.acer.edu.au/cgi/ p103

[23] There is moderate evidence that effective numeracy intervention at any year level should include a proportion of time devoted to practising fluency retrieval of basic facts Gersten et al 2009 http://ies.ed.gov/ncee/wWc/pdf/practice_guides/rti_math_pg_042109.pdf p37

[24] http://www.hanoverresearch.com/media/Best-Practices-in-Math-Interventions.pdf p16;Effect size based on a study by the program's designers but recognised as methodologically sound

[25] http://www.hanoverresearch.com/media/Best-Practices-in-Math-Interventions.pdf p17 Effect size based on a study by the program's designers but recognised as methodologically sound; effect size diminishes as questions require more advanced applications

Bookmarks

Model for Stages of Early Arithmetical Learning (SEAL)[26]:

Stage 0	Emergent Counting	Cannot count visible items. The child either does not know the number words or cannot co-ordinate the number words with items.
Stage 1	Perceptual Counting	Can count visible items only
Stage 2	Figurative Counting	Can count invisible items, but starts from one instead of counting-on.
Stage 3	Initial number Sequence	Child uses counting-on rather than counting from one to solve addition or missing addend tasks (eg: 6+x=9). The child may use a counting-back strategy (counting back-from or counting back-to) to solve remove item tasks (eg: 17-3 as 16,15,14 answer = 14)
Stage 4	Immediate Number Sequence	The child uses more efficient count-down-from and count-down-to strategies.
Stage 5	Facile Number sequence	Can use non-counting-by-one strategies, such as doubles, add through ten, compensation, using a known result, subtraction as the inverse of addition, awareness of 10 in a 'teen' number

[26] http://www.edubuzz.org/numeracyleadership/wp-content/blogs.dir/2343/files/2015/05/Presentation-2-Stages-of-Arithmetical-Learning.pdf

References

ACER 2015 *Literacy and Numeracy Interventions in the Early Years of Schooling: A Literature Review* http://research.acer.edu.au/cgi/viewcontent.cgi?article=1019&context=policy_analysis_misc

Hanover Research 2014 *Best Practices in Math Interventions* http://www.hanoverresearch.com/media/Best-Practices-in-Math-Interventions.pdf

Hattie 2009 Visible Learning London & New York Routledge

http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/misunderstandings.aspx

http://www.scootle.edu.au/ec/viewing/S7059/pdf/tls48_targeted_early_numeracy.pdf

http://www.edubuzz.org/numeracyleadership/wp-content/blogs.dir/2343/files/2015/05/Presentation-2-Stages-of-Arithmetical-Learning.pdf

Institute of Education Sciences US Department of Education 2009 *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools What Works Clearing House* http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf

MUSEC Briefing May 2011 *Schema Based Instruction for Mathematics Word Problem-solving* Issue 27

Wendling B & Mather N 2009 *Essentials of evidence-based academic Interventions* John Wiley & Sons

Wright R et al 2006 *Teaching Number – Advancing Children's Skills and Strategies* London Paul Chapman Publishing https://books.google.com.au/books?id=tK_xJbSyQMIC&pg=PA9&lpg=PA9&dq=Steffe+and+colleagues+counting+stages&source=bl&ots=CWhqsYl2ul&sig=5mv3E7eq-HelAp7BJmMj720Eqxo&hl=en&sa=X&ved=0ahUKEwj_4PyF6tjOAhUJlJQKHQtsCEAQ6AEIHTAA#v=onepage&q=Steffe%20and%20colleagues%20counting%20stages&f=false